Second-order (P-Δ and P-δ) Effects

Overview

Second-order effects in beam-columns are investigated. Results obtained using rigorous second-order elastic analyses are compared with approximate moment amplification factors. Both P-Δ and P-δ effects are observed.

Learning Objectives

- Employ rigorous second-order elastic analysis to determine required strengths in beam-columns.
- Observe the relationship between the amount of moment amplification in a member and its ratio of axial force to elastic critical load.
- Investigate the impact that single or double (reverse) curvature bending has on the degree of moment amplification.
- Compute and assess the moment amplification factor B_1 defined in Appendix 8 of the AISC *Specification for Structural Steel Buildings* (2016).

Method

Part I.

Prepare a computational model of the beam-column shown in Fig. 1. Assume the member is a W14x48 (A992) oriented for major-axis bending and fully braced out-of-plane. Varying the amount of axial force P, employ rigorous second-order elastic analysis to complete Tables 1a, 1b, and 1c for the three M_1/M_2 ratios given. In addition to computing the amount of amplification observed $|M_{max}|_{span}/|M_{max}|_{end}$, calculate the corresponding moment amplification factor B_1 defined in Eq. A-8-3 of Appendix 8 of the AISC AISC *Specification for Structural Steel Buildings* (2016).
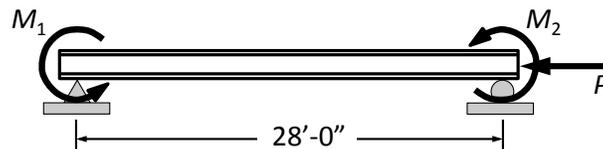

Figure 1.

Part II.

Prepare computational models of the beam-columns shown in Fig. 2. Again, assume the members are W14x48 (A992) oriented for major-axis bending and fully braced out-of-plane. Perform the rigorous second-order elastic analyses required to complete Tables 2a and 2b.
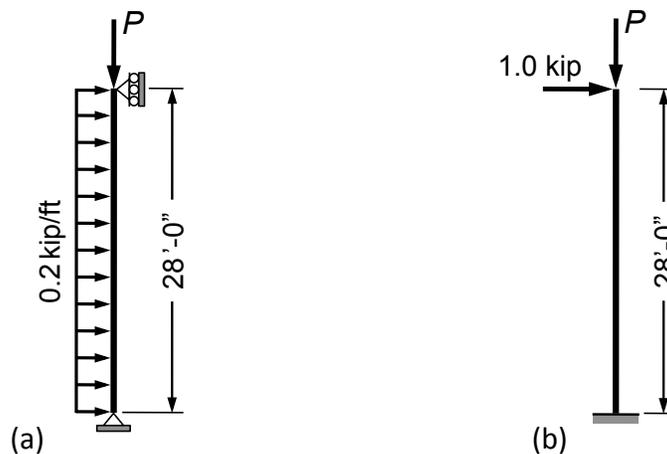

Figure 2.

Hints:
1) Suggested units are kips, inches, and ksi.
2) Do not include initial imperfections such as member-of-straightness.
3) Do not include the self-weight of the member.
4) Perform two-dimensional (planar frame) analyses.

MASTAN2 Details

Per Fig. 3, the following suggestions are for those employing MASTAN2 to complete the above studies:
✓ In several cases, the maximum moment may occur at any point along the member span. Thus, it would be ideal to have an element end (node) at every location. Given that this is impossible, subdividing the member into 10 elements should provide an adequate number of sampling points.
✓ By leaving Ayy = inf and Azz = inf when defining the section properties, shear deformations are neglected.
✓ For the second-order elastic analyses, use the following options:
- Planar frame analysis type
- Solution type: Predictor-Corrector for $P/P_e < 0.6$ and Work-Control for $P/P_e \geq 0.6$
- Load increment size of 0.1
- Maximum number of increments set to 100
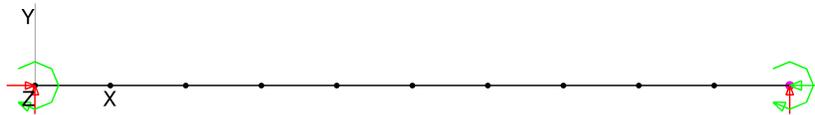- Maximum applied load ratio set to 1.0

Figure 3. MASTAN2 model.

Questions
1) Prepare a single plot that illustrates the relationship between P/P_e (abscissa) and the amount of moment amplification (ordinate) observed in Tables 1a, 1b, and 1c. For each of the three moment gradient M_1/M_2 cases, provide two curves; one for the computational ratio $|M_{max}|_{span}/|M_{max}|_{end}$ and the other based on the B_1 factor. Comment on the accuracy of the approximate moment amplification B_1 factor. For which moment gradient M_1/M_2 case does the approximate B_1 factor provide the most accurate results? With this in mind, which term in the B_1 equation is most likely the greatest source of error?
2) From the above plot and the results recorded in Tables 2a and 2b, please discuss the relationship between the amount of moment amplification in a member and its ratio of axial force to elastic critical load.
3) Using results obtained from only the computational analyses performed in Part I, compare the three moment gradient cases. Please comment on the degree to which bending in single or double (reverse) curvature influences moment amplification.
4) Compare the above plot with the plot shown in Fig. C-A-8.3 of the Commentary to the AISC *Specification for Structural Steel Buildings* (2016). In general, are your results in agreement with those presented?
5) Given that the elastic flexural buckling strength P_e has two values, one for major-axis bending and one for minor-axis bending, provide a rule of thumb for deciding which to use in predicting the expected amount of moment amplification.
6) In Tables 2a and 2b, are the amplification (2nd-order/1st-order) ratios recorded approximately consistent for the recorded moments and deflections? Comment on this observation.
7) To confirm the accuracy of the analysis software you employed, please compare the recorded Tables 2a and 2b with those presented in Figs. C-C2.2 and C-C2.3 of the Commentary to the AISC *Specification for Structural Steel Buildings* (2016). Did your analyses include or neglect shear deformations? Why does AISC provide two sets of results (including and excluding shear deformations)? With this in mind, does

the analysis software you employed provide an adequate degree of accuracy to meet the requirements defined by the benchmark problems and solutions given in the AISC Commentary?

More Fun with Computational Analysis!

Explore a structural system, such as a portal frame or the frame presented in Learning Module 2, and compare first-order and second-order elastic analysis results. Be sure to study deflections and internal moment, shear, and axial force distributions.

Additional Resources

MS Excel spreadsheet: *7_SecondOrderEffects.xlsx*

LM7 Tutorial Video [6 min]:

http://www.youtube.com/watch?v=xxAW2AGyr3k

AISC *Specification for Structural Steel Buildings and Commentary* (2016):

https://www.aisc.org/publications/steel-standards/ - 30666

MASTAN2 software:

http://www.mastan2.com/

Table 1a.

$M_1/M_2 = +0.75$		$M_1 = 150$ kip-in	$M_2 = 200$ kip-in		$C_m =$	AISC Eq. A-8-3
$P_e =$	kips	2nd-Order Elastic Analysis			AISC	Percent
P/P_e	P kips	$\|M_{max}\|_{span}$ kip-in	$\|M_{max}\|_{end}$ kip-in	Ratio $\|M_{max}\|_{span}/\|M_{max}\|_{end}$	Eq. A-8-3 B_1	Diff %
0.0	0.00	200	200	1.00	1.00	0.00
0.1			200			
0.2			200			
0.3			200			
0.4			200			
0.5			200			
0.6			200			
0.7			200			
0.8			200			
0.9			200			

$\|M_{max}\|_{span}$ = magnitude of maximum moment along span including the member ends

Table 1b.

$M_1/M_2 = 0.0$		$M_1 = 0$ kip-in	$M_2 = 200$ kip-in		$C_m =$	AISC Eq. A-8-3
$P_e =$	kips	2nd-Order Elastic Analysis			AISC	Percent
P/P_e	P kips	$\|M_{max}\|_{span}$ kip-in	$\|M_{max}\|_{end}$ kip-in	Ratio $\|M_{max}\|_{span}/\|M_{max}\|_{end}$	Eq. A-8-3 B_1	Diff %
0.0	0.00	200	200	1.00	1.00	0.00
0.1			200			
0.2			200			
0.3			200			
0.4			200			
0.5			200			
0.6			200			
0.7			200			
0.8			200			
0.9			200			

$\|M_{max}\|_{span}$ = magnitude of maximum moment along span including the member ends

Table 1c.

$M_1/M_2 = -0.75$		$M_1 = -150$ kip-in	$M_2 = 200$ kip-in		$C_m =$	AISC Eq. A-8-3
$P_e =$	kips	2nd-Order Elastic Analysis			AISC	Percent
P/P_e	P kips	$\|M_{max}\|_{span}$ kip-in	$\|M_{max}\|_{end}$ kip-in	Ratio $\|M_{max}\|_{span}/\|M_{max}\|_{end}$	Eq. A-8-3 B_1	Diff %
0.0	0.00	200	200	1.00	1.00	0.00
0.1			200			
0.2			200			
0.3			200			
0.4			200			
0.5			200			
0.6			200			
0.7			200			
0.8			200			
0.9			200			

$\|M_{max}\|_{span}$ = magnitude of maximum moment along span

Table 2a.

Axial Force, P (kips)	0	150	300	450
Ratio of P / P_e				
M_{mid} (kip-in)				
$M_{mid} / (wL^2/8)$				
Δ_{mid} (in.)				
$\Delta_{mid} / (5wL^4/384EI)$				

Table 2b.

Axial Force, P (kips)	0	100	150	200
Ratio of P / P_e				
M_{base} (kip-in)				
$M_{base} / (1.0 \times L)$				
Δ_{tip} (in.)				
$\Delta_{tip} / (1.0 \times L^3/3EI)$				