Lateral-Torsional Buckling of Beams with Moment Gradient

Overview
The influence of moment gradient on the lateral-torsional buckling capacity of compact wide-flange beams is studied. Equivalent uniform moment factors C_b back-calculated from computational analyses are compared with values computed using two longstanding equations, one appearing in Chapter F of the AISC *Specification for Structural Steel Buildings* (2016) and the other in the commentary to the Specification.

Learning Objectives
- Observe the effect that a non-uniform moment distribution has on the lateral-torsional buckling capacity of compact wide-flange beams.
- Back-calculate equivalent uniform moment factors C_b from results of elastic critical load analyses.
- Compute values for C_b using two well-established equations and compare them with results from computational analyses.
- Investigate several different possibilities of moment gradient, including linear, bi-linear, and parabolic.
- Study the impact of providing an interior brace point.

Method
Prepare a computational model of a W24x68 (A992) beam with an unbraced length of 24-ft. For each of the three studies shown below, perform two analyses:
- First-order elastic analysis. Record the magnitude of the internal moments at the three quarter points ($M_{L/4}$, $M_{L/2}$, $M_{3L/4}$) and the maximum moment from a plot of the moment diagram.
- Elastic Critical Load Analysis. Record the magnitude of the largest internal moment from a plot of the moment diagram. Of course, this value should be the product of the resulting applied load ratio and the maximum moment recorded from the above first-order elastic analysis.

Studies:
1) Linear moment distribution (Fig. 1a): Complete Table 1 by subjecting the beam to the various combinations of major-axis end moments given in the table. Note that the maximum moment is M_1. Show the two columns of computed equivalent uniform moment factors as curves on a single plot, with the abscissa being the ratio M_2/M_1 and the ordinate C_b.
2) Bi-linear moment distribution (Fig. 1b): Complete Table 2 by subjecting the beam to a concentrated force P located at the various x values given in the table. Show the two columns of computed equivalent uniform moment factors as curves on a single plot, with the abscissa being the ratio x/L and the ordinate C_b.
3) Parabolic moment distribution (Fig. 1c). Subject the beam to the uniformly distributed loads defined in Table 3 and record the desired internal moments.
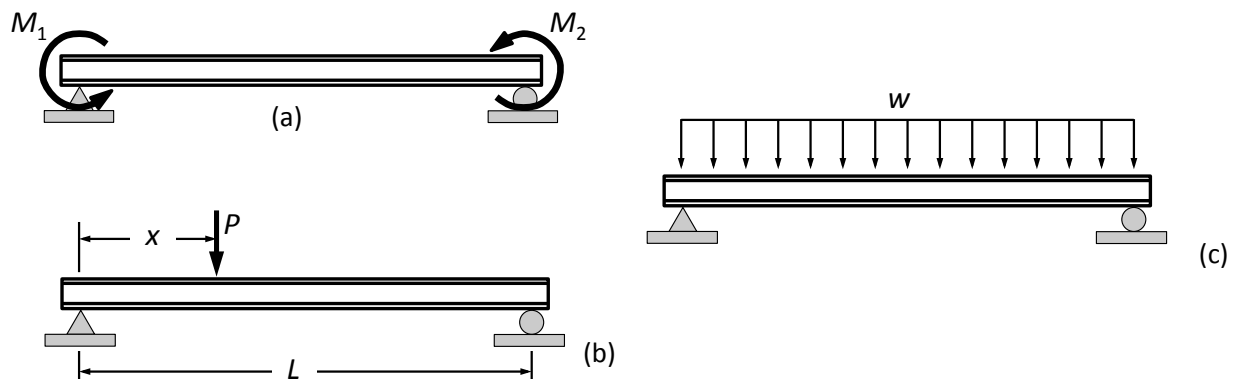

Figure 1.

Hints:
1) Suggested units are kips, inches, and ksi.
2) For Elastic Critical Load analyses, the models need not include initial imperfections such as member-of-straightness.
3) 3-Dimensional (space frame) analyses are required. Support conditions at the member ends should include all translation degrees of freedom restrained with the exception of longitudinal translation at one end of the member. Torsional degrees of freedom (rotation about the longitudinal axis) at both member ends should also be restrained. Warping should be modeled as continuous along the span length and free at the member ends.
4) Do not include the self-weight of the member.

Table 1.

First, compute $M_{cr} = \frac{\pi}{L}\sqrt{EI_yGJ + I_yC_w(\pi E/L)^2}$ =							(kip-in)		
			1st-Order Elastic Analysis			AISC Eq. F1-1	Elastic Critical Load Analysis		% diff. C_b's
M_2/M_1	M_1 (kip-in)	M_2 (kip-in)	$\|M_{L/4}\|$ (kip-in)	$\|M_{L/2}\|$ (kip-in)	$\|M_{3L/4}\|$ (kip-in)	C_b	$\|M_{max}\|$ (kip-in)	$C_b = \|M_{max}\|/\|M_{cr}\|$	
-1.0	1000	-1000							
-0.8	1000	-800							
-0.6	1000	-600							
-0.4	1000	-400							
-0.2	1000	-200							
0.0	1000	0							
+0.2	1000	200							
+0.4	1000	400							
+0.6	1000	600							
+0.8	1000	800							
+1.0	1000	1000							

Table 2.

First, compute $M_{cr} = \frac{\pi}{L}\sqrt{EI_yGJ + I_yC_w(\pi E/L)^2}$ =							(kip-in)		
		1st-Order Elastic Analysis				AISC Eq. F1-1	Elastic Critical Load Analysis		% diff. C_b's
P (kip)	x/L	$\|M_{L/4}\|$ (kip-in)	$\|M_{L/2}\|$ (kip-in)	$\|M_{3L/4}\|$ (kip-in)	$\|M_{max}\|$ (kip-in)	C_b	$\|M_{max}\|$ (kip-in)	$C_b = \|M_{max}\|/\|M_{cr}\|$	
100	0.125								
100	0.250								
100	0.375								
100	0.500								
100	0.625								
100	0.750								
100	0.875								

Table 3.

First, compute $M_{cr} = \frac{\pi}{L}\sqrt{EI_y GJ + I_y C_w (\pi E/L)^2}$ =							(kip-in)	
	1st-Order Elastic Analysis				AISC Eq. F1-1	Elastic Critical Load Analysis		% diff. C_b's
w (kip/in)	$\|M_{L/4}\|$ (kip-in)	$\|M_{L/2}\|$ (kip-in)	$\|M_{3L/4}\|$ (kip-in)	$\|M_{max}\|$ (kip-in)	C_b	$\|M_{max}\|$ (kip-in)	$C_b =$ $\|M_{max}\|/\|M_{cr}\|$	
0.10								
0.30								

MASTAN2 Details

Per Fig. 2, the following suggestions are for those employing MASTAN2 to calculate the above computational strengths:

✓ Subdivide the member into 8 elements.
✓ Warping resistance to torsion can be modeled along the member span by using MASTAN2's option under Geometry > Define Connections > Torsion and setting the warping restraint at both ends of all elements to "Continuous."
✓ Because it may be difficult to observe twist when working with one-dimensional line elements, it is suggested that a few additional elements be added at the mid-span of the member that are perpendicular to its longitudinal axis. Given that these elements should not resist any of the applied moments, their section properties only need to be non-zero.
✓ Only one mode is needed in the Elastic Critical Load (eigenvalue) Analyses; be sure to complete Space Frame analyses.
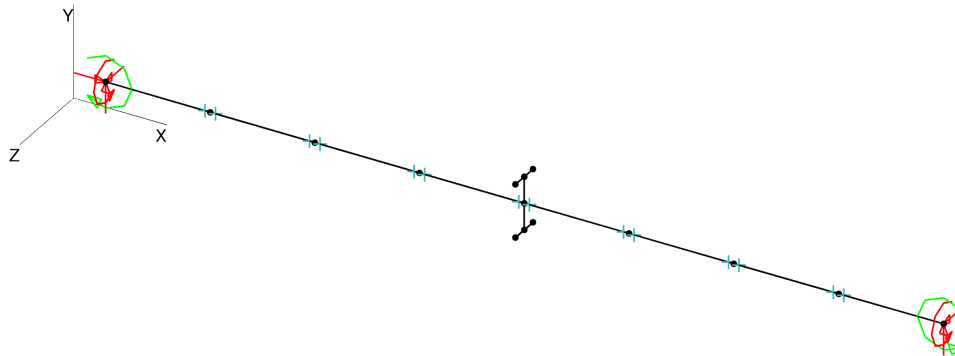

Figure 2. MASTAN2 model of Fig. 1a.

Questions

1) For each of the three moment distributions, comment on the accuracy of AISC Eq. F1-1. Is the use of this equation conservative, overly conservative, or unconservative, when compared to the results of an elastic critical load analysis? Please qualify your response based on the specific cases investigated.
2) For many editions, the AISC Specifications employed Eq. C-F1-1; see Commentary to the AISC *Specification for Structural Steel Buildings* (2016). Use this equation to compute C_b's for the three moment distributions studied and comment on its accuracy. Suggest a rule of thumb for when it may or may not give adequate results.
3) In studying the beam with a uniformly distributed load, were the C_b factors a function of the magnitude of the load w? Provide an explanation to your response.
4) Repeat a portion of the above studies (e.g. $M_2/M_1 = +1.0$, P at $x/L = 0.5$, and $w = 0.1$) by providing a lateral brace (i.e., restraining out-of-plane lateral movement and twist about the length axis of the member) at the mid-span of the beam. Note that C_b factors should be computed for the span to the left of the brace and for the span to the right. Compare these results with previous results that did not include a mid-span

brace. What do you observe? For the specific case of $M_2/M_1 = +1.0$ in Study 1, can one conclude that an inflection point in a beam without a brace is equivalent to a beam with brace at the location of this inflection point (be sure to compare the results of the two critical load analyses, one with the mid-span brace and one without)? Justify your response.

More Fun with Computational Analysis!

1) Use the results of Elastic Critical Load analyses to back-calculate C_b factors for several of the cases shown in Table 3-1 of the AISC *Steel Construction Manual* (2017). How well do these values compare to those provided in the table? Note that the brace(s) shown can be modeled in the analysis by restraining the out-of-plane translation and twist about the length axis of the member at the brace point(s).

2) Modify the investigation of the beam with a uniformly distributed load (Study 3) by including equal and opposite concentrated moments of $M = \beta wL^2/12$ at the member ends, with β varying from 0 to 2.0. Note that the direction of the end moments are defined so that an equivalent fixed ends condition occurs when $\beta = 1.0$. Prepare a plot with the abscissa being the ratio β and the ordinate C_b.

3) Repeat all portions of the above studies by providing a lateral brace (out-of-plane restraint) at the mid-span of the beam. Note that C_b factors should be computed for the span to the left of the brace and for the span to the right.

4) Repeat the previous problem also including a torsional brace. Are there significant benefits?

5) Repeat the original study also including some amount of axial force (e.g. $P = 0.2P_y = 0.2A_gF_y$). Extend this study to include a range of axial forces.

6) Perform a study that employs the second-order inelastic analysis concepts presented in Learning Module Number 4. How do C_b factors back-calculated from second-order inelastic analyses compare with the corresponding elastic C_b's? Do not employ Inelastic Critical Load analyses.

Additional Resources

MS Excel spreadsheet: *5_LTBofBeamswithMomentGradient.xlsx*

MASTAN2 – LM5 Tutorial Video [13 min]:
> http://www.youtube.com/watch?v=rrgfq1hRSaU

MASTAN2 - How to include warping resistance [1 min]:
> http://www.youtube.com/watch?v=ttoVaiEnn0M

AISC *Specification for Structural Steel Buildings and Commentary* (2016):
> https://www.aisc.org/publications/steel-standards/ - 30666

MASTAN2 software:
> http://www.mastan2.com/