Learning Module Number 3

Effective Length *K*-factors for Frame Members

Overview

Effective length *K*-factors for compression members in frames are investigated. In addition to studying cases of sidesway inhibited and unhibited, elastic and inelastic buckling is explored. Computational results are compared with alignment chart methods.

Learning Objectives

- Observe the elastic and inelastic stability behavior of frames under sidesway inhibited and uninhibited conditions.
- Back-calculate effective length *K*-factors from results of elastic and inelastic critical load analyses.
- Use the alignment charts to obtain effective length *K*-factors assuming elastic and inelastic column behavior. Assess the applicability of the assumptions made in developing the alignment charts.
- Compare results from the alignment chart methods and computational methods.

Method

Prepare a computational model of the one-story, one-bay planar frame shown in Fig. 1. All members are fabricated from A992 steel and all connections are fully restrained moment connections. Apply a vertically downward 1-kip load to the top of each column. Be sure to orient the columns for minor-axis bending and the beams for major-axis bending.

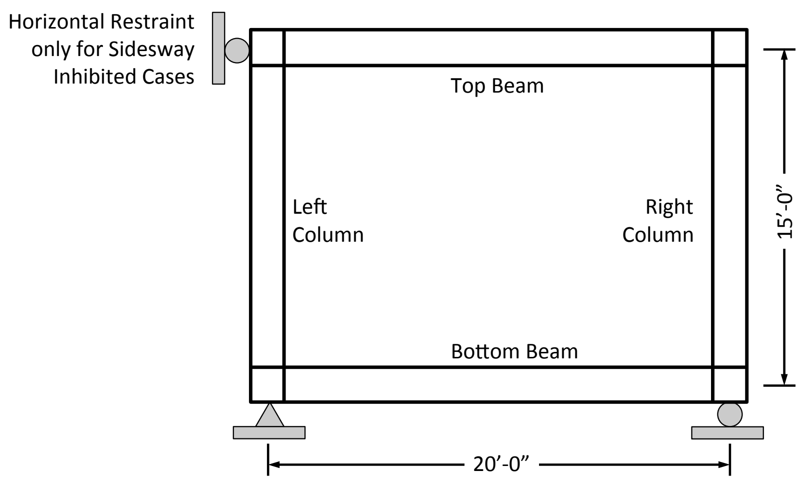

Figure 1.

Complete Table 1 by performing planar frame (2D) elastic and inelastic critical load analyses[1] and computing the first buckling mode. For each case, record the axial force P_{cr} in the column and also note the general form of the buckled shape focusing on the degree of bending in the beams and the approximate location of the inflection points in the columns. To gain a full appreciation of the bending deformations, try to avoid resetting the scale of the deflected shape for each analysis.

Using the column forces P_{cr} in Table 1, back-calculate effective length *K*-factors and record them in Table 2. For an elastic critical load analysis $K = \pi/L\sqrt{EI/P_{cr}}$, where L = length of column, E = elastic modulus, and I is the column's moment of inertia about its bending axis. For an inelastic analysis, use the same expression except replace the elastic modulus E with the tangent modulus E_t, where $E_t = \tau E$ with $\tau = 4(P_{cr}/P_y)(1 - P_{cr}/P_y)$ for $P_{cr}/P_y > 0.5$, and $\tau = 1$ for $P_{cr}/P_y \leq 0.5$, all with $P_y = AF_y$.

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[1] An elastic critical load analysis is also referred to as an elastic buckling or eigenvalue analysis.

Complete Table 2 by using the alignment charts, which are given in Figures C-A-7.1 and C-A-7.2 of the Commentary on the AISC *Specification for Structural Steel Buildings* (2016), to compute elastic and inelastic effective length *K*-factors. Recall that $G = \sum(EI/L)_{column}/\sum(EI/L)_{beam}$ and when accounting for inelasticity, replace $(EI/L)_{column}$ with $(E_t I/L)_{column}$ using the above expression for E_t.

Hints:
1) Suggested units are kips, inches, and ksi.
2) Do not include the self-weight of the member.

Table 1.

Member Sizes					Sidesway Inhibited		Sidesway Uninhibited	
Case	Left Column	Right Column	Top Beam	Bottom Beam	Elastic P_{cr}	Inelastic P_{cr}	Elastic P_{cr}	Inelastic P_{cr}
1	W10x33	W10x33	W12x14	W12x14				
2	W10x33	W10x33	W24x68	W24x68				
3	W10x33	W10x33	W24x68	W12x14				
4	W10x33	W10x33	W12x14	W24x68				

Table 2.

	Effective Length *K*-Factors						
	Case	Elastic Critical Load	Elastic Alignment Charts	Percent Difference (%)	Inelastic Critical Load	Inelastic Alignment Charts	Percent Difference (%)
Sidesway Inhibited	1						
	2						
	3						
	4						
Sidesway Uninhibited	1						
	2						
	3						
	4						

MASTAN2 Details

Per Fig. 1, the following suggestions are for those employing MASTAN2 to calculate the above computational strengths:

- ✓ Subdivide each column into 8 elements. There is no need to subdivide the beams because they resist an inconsequential amount of axial force
- ✓ In all computational analyses, the failure load will be the product of the applied force (1-kip) and the resulting Applied Load Ratio.

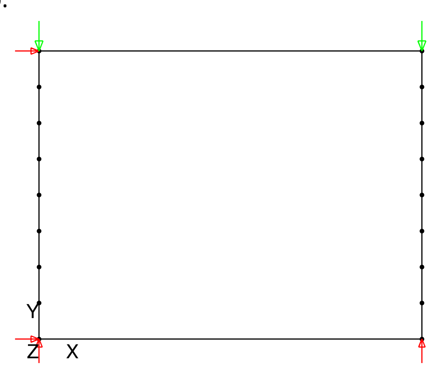

Figure 1. MASTAN2 model (sidesway inhibited).

Questions

1) Using the data recorded in Table 1, how does the relative bending stiffness of the beams to the columns impact the buckling strength of the frame? Explain why this relationship is not as pronounced for the inelastic analyses as it is for the elastic analyses?
2) How well do the effective length *K*-factors compare for the computational and alignment chart methods?
3) Would it be more or less conservative to always use effective length *K*-factors calculated from an elastic analysis? For this problem, is it worth the extra effort to compute inelastic *K*-factors? Justify your response.
4) How much does a 5% difference in an effective length *K*-factor have on the nominal compressive strength as defined by the AISC Specification (provide two answers, one for inelastic buckling given by Eq. E3-2 and one for elastic buckling Eq. E3-3)? Conversely, what is the tolerance (+/- percent) on an effective length *K*-factor that would correspond to a 5% difference in compressive strength (again, provide tolerances for both Eqs. E3-2 and E3-3)? Are all of these values generic or specific to this problem of investigating a 15-ft long W10x33 column subject to minor axis bending? Justify your response.
   - Referring to the results in Table 2, is it conservative or unconservative to simply employ *K* = 1 in calculating the compressive strength of the columns? You may want to provide separate responses for the sidesway inhibited and uninhibited cases.
   - The alignment charts are based on assumptions of many idealized conditions that are defined in most steel design textbooks and Appendix 7 of the Commentary on the AISC *Specification for Structural Steel Buildings* (2016). Carefully review these assumptions and comment on their applicability to this frame study.

More Fun with Computational Analysis!

Calculating separate effective length *K*-factors for the two columns, repeat the above study for the following cases.

1) A simple (pinned) connection at the right end of the top and bottom beams.
2) The right column oriented for major-axis bending.
3) A 1-kip force only on the top of the right column.
4) A pinned connection at the right end of the top and bottom beams and a 1-kip force only on the top of the right column.

Additional Resources

MS Excel spreadsheet: *3_EffectiveLengthKfactorsforFrameMembers.xlsx*

MASTAN2 – LM3 Tutorial Video [10 min]:
http://www.youtube.com/watch?v=0S9XI3oVWws

MASTAN2 - How to re-orient elements for minor-axis bending [2 min]:
http://www.youtube.com/watch?v=kqcPlDvw95U

AISC *Specification for Structural Steel Buildings and Commentary* (2016):
[https://www.aisc.org/publications/steel-standards/ - 30666](https://www.aisc.org/publications/steel-standards/)

MASTAN2 software:
http://www.mastan2.com/