Learning Module Number 4

Factors Influencing the Strength of Flexural Members

Overview

Using computational analysis as a virtual laboratory, the main factors that impact the major-axis flexural strength of steel wide-flange sections with compact elements are investigated. These factors include unbraced length, partial yielding accentuated by the presence of residual stresses, and initial imperfections in geometry (out-of-straightness). Strength limit states defined by full yielding of the cross-section (plastic hinge) and elastic/inelastic lateral torsional buckling are studied. Computed strengths are presented in the form of beam strength curves, which are further compared with the corresponding nominal strength curve defined in Chapter F of the AISC *Specification for Structural Steel Buildings* (2022).

Learning Objectives

- Recognize limitations of the theoretical elastic lateral-torsional buckling solution.
- Prepare beam curves that plot member unbraced length versus flexural strength.
- Observe the impact that partial yielding (accentuated by the presence of residual stresses) and initial imperfections in geometry have separately and collectively on the elastic and inelastic lateral-torsional buckling strength of a flexural member.
- Compare results to the AISC beam strength curve.

Method

Employing theory, computational analyses, and the AISC Specification, prepare a series of curves that show the major-axis flexural strength of a W14x53 (A992 steel). Plot all curves on the same figure with unbraced length L as the abscissa and the ratio of the calculated flexural strength to the plastic moment capacity M_n/M_p as the ordinate. In all cases, the member is subject to a uniform major-axis bending. Unbraced lengths should include L = 1.2, 3.4, **6.8**, 10.6, 14.5, 18.4, **22.2**, 26.6, and 31.1 ft. It is suggested that pairs of students compute following results, with both investigating L = 18.4 ft and then one student investigating L = 1.2, 6.8, 14.5, and 26.6 ft., and the other L = 3.4, 10.6, 22.2, and 31.1 ft.; all curves and discussion should be prepared individually.

Curves should include the following cases:

1) Full plastic moment capacity (Easy one! M_n/M_p = 1.0 for all L).
2) Theoretical elastic lateral-torsional buckling strength, $M_{cr} = \frac{\pi}{L}\sqrt{EI_yGJ + I_yC_w(\pi E/L)^2}$.
3) Nominal strength M_n as defined by the AISC Specification (Eqs. F2-1, F2-2, and F2-3).
4) Computational strength that does not account for initial imperfection and partial yielding/residual stresses.
5) Computational strength that includes a maximum initial imperfection of L/1000 (out-of-plane out-of-straightness at mid-span), but does not account for partial yielding/residual stresses.
6) Computational strength that accounts for partial yielding/residual stresses, but does not include an initial imperfection.
7) Computational strength that includes maximum initial imperfection of L/1000 and accounts for partial yielding/residual stresses.
8) Computational strength calculated by an Elastic Critical Load (eigenvalue) analysis.
9) Computational strength calculated by a sophisticated nonlinear finite element analysis (ADINA) that employs shell elements, which produced the following results

L (ft)	1.2	3.4	**6.8**	10.6	14.5	18.4	**22.2**	26.6	31.1
M_n/M_p	0.993	0.965	0.911	0.811	0.704	0.605	0.519	0.440	0.382

Add two vertical lines to the plot at L_p and L_r as defined by the AISC Specification (Eqs. F2-5 and F2-6) and one horizontal line at $M_r/M_p = 0.7S_xF_y/Z_xF_y = 0.7S_x/Z_x$.

Hints:

1) Suggested units are kips, inches, and ksi.

2) Maintain two computational models for each unbraced length *L*, one without imperfections and one with imperfections. The initial imperfection of *L*/1000 is in the out-of-plane direction.
3) 3-Dimensional (space frame) analyses are required. Support conditions at the member ends should include all translation degrees of freedom restrained with the exception of longitudinal translation at one end of the member. Torsional degrees of freedom (rotation about the member's longitudinal axis) at both member ends should also be restrained. Warping should be modeled as continuous along the span length and free at the member ends.
4) Member ends should be loaded with equal and opposite major-axis bending moments.
5) Do not include the self-weight of the member.

MASTAN2 Details

Per Fig. 1, the following suggestions are for those employing MASTAN2 to calculate the above computational strengths:

- ✓ For each unbraced length, prepare two parallel compression members; one will include the initial imperfection.
- ✓ Subdivide the members into 8 elements.
- ✓ Initial imperfections (as needed) can be included by either extensive use of the *Move Node* option, or much more easily by "permanently bending" the member through the combined use of either a buckling analysis or lateral load analysis, and MASTAN2's post-processing option *Results-Update Geometry*.
- ✓ Because it may be difficult to observe twist when working with one-dimensional line elements, it is suggested that a few additional elements be added at the mid-span of the member that are perpendicular to its longitudinal axis. Given that these elements should not resist any of the applied moments, their section properties only need to be non-zero.
- ✓ In all computational analyses, use applied major-axis bending moments of 1000 kip-in. The failure moment will be the product of this moment (1000 kip-in) and the resulting Applied Load Ratio.
- ✓ Warping resistance to torsion can be modeled along the member span by using MASTAN2's option under Geometry > Define Connections > Torsion and setting the warping restraint at both ends of all elements to "Continuous."
- ✓ With the exception of the eigenvalue analyses, employ second-order inelastic analyses with:
    - Space frame analysis type
    - Predictor-corrector solution scheme
    - Load increment size of 0.01
    - Maximum number of increments set to 1000
    - Maximum applied load ratio set to 10
    - Modulus set to either *E* (no partial yielding/residual stresses) or E_{tm} (account for partial yielding/residual stresses)
- ✓ If the analysis pauses and indicates that a significant change in deformations is detected, this means that a plastic mechanism has formed. There is no need to continue the analyses.
- ✓ Only one mode is needed in the Elastic Critical Load (eigenvalue) Analyses; be sure to complete Space Frame analyses.

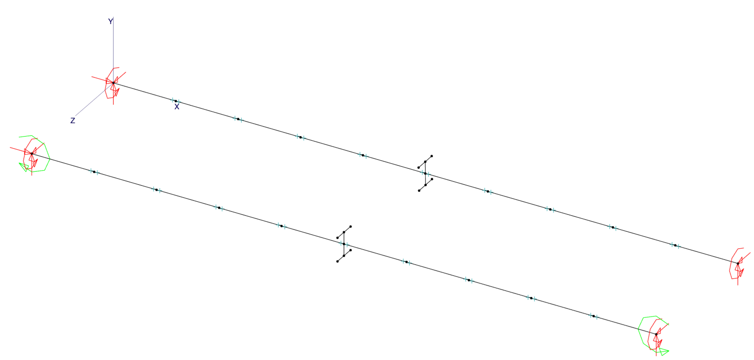

Figure 1. MASTAN2 model.

Questions

1) With all curves included and labeled, submit two plots with (i) the maximum ordinate set to the maximum M_n/M_p obtained, and (ii) the maximum ordinate set to 1.1. What is causing the first plot to have such an extreme strength value? Is this value realistic?
2) Is the theoretical elastic lateral-torsional buckling strength curve in agreement with the Elastic Critical Load curve? For what range of unbraced lengths are these curves even remotely realistic and for what range are they unacceptable? Justify your response.
3) Using the computational results, define a range of unbraced lengths for which partial yielding/residual stresses appear to have the greatest impact.
4) Likewise, provide a range of unbraced length in which initial imperfections appear to have the greatest impact.
5) Comment on the accuracy of the AISC beam curve (Eqs. F2-1, F2-2, and F2-3) for this particular shape.
6) Describe what the minor-axis beam strength curve would look like if it were added to this plot.
7) Which curves would change if a different wide-flange section size is investigated? Justify your response.

More Fun with Computational Analysis!

1) Repeat the above exercise for the following cases.
   a. Neglecting the effects of warping resistance (warping free at all locations along the span). In general, are their benefits to modeling warping effects and if so, qualify your response in relation to the unbraced length?
   b. Warping defined as fixed at the member ends and continuous along the span length. Note that the AISC curve (case 3) does not change. For this condition, sophisticated nonlinear finite element analysis (case 9) of a W14x53 provides

L (ft)	1.2	3.4	6.8	10.6	14.5	18.4	22.2	26.6	31.1
M_n/M_p	0.996	0.980	0.949	0.902	0.853	0.784	0.711	0.603	0.518

   c. Investigate the major-axis flexural strength of a W24x68 (A992 steel).
2) Have each student in the class investigate a different wide flange section and/or warping end restraint (free/fixed). Prepare a composite of beam curves that include each student's second-order inelastic analysis results that account for both initial imperfection and partial yielding/residual stresses (case 7). Compare this collection of curves with the AISC curve (case 3).

Additional Resources

MS Excel spreadsheet: *4_StrengthOfFlexuralMembers.xlsx*
MASTAN2 – LM4 Tutorial Video [15 min]:
  http://www.youtube.com/watch?v=BUXRDDVRwmA
MASTAN2 - How to include warping resistance [1 min]:
  http://www.youtube.com/watch?v=ttoVaiEnn0M
MASTAN2 - How to include an initial imperfection (member out-of-straightness) [4 min]:
  http://www.youtube.com/watch?v=v3ON1faDSZo
MASTAN2 - How to account for partial yielding accentuated by residual stresses [1 min]:
  http://www.youtube.com/watch?v=m8ZXM02Cbu4
AISC *Specification for Structural Steel Buildings and Commentary* (2022):
  https://www.aisc.org/publications/steel-standards/aisc-360/
MASTAN2 software:
  http://www.mastan2.com/

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Table 1

Length (ft.)	Length (in.)	Plastic Capacity (M_n=M_p)		Theoretical LTB (M_n=M_{cr})		AISC		No L/1000 + No partial yielding		L/1000 + No partial yielding	
		M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p
1.2			1.00								
3.4			1.00								
6.8			1.00								
10.6			1.00								
14.5			1.00								
18.4			1.00								
22.2			1.00								
26.6			1.00								
31.1			1.00								

Length (ft.)	Length (in.)	No L/1000 + partial yielding		L/1000 + partial yielding		Elastic critical		ADINA	
		M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p	M_n (kip-in)	M_n/M_p
1.2								4325	0.993
3.4								4203	0.965
6.8								3967	0.911
10.6								3532	0.811
14.5								3066	0.704
18.4								2635	0.605
22.2								2260	0.519
26.6								1916	0.440
31.1								1664	0.382