

## Learning Module Number 1

### Elastic Column Buckling and the Effect of End Restraint

#### Overview

Using a critical load analysis, the elastic flexural buckling strength of a column with various degrees of end restraint is investigated.

#### Learning Objectives

- Employ computational software to verify the theoretical elastic column buckling solutions obtained from the use of differential equations.
- Observe how the load carrying capacity and buckled shape vary with different types of end restraint.
- Confirm the theoretical effective length  $K$ -factors that appear in Table C-A-7.1 of the Commentary on the *AISC Specification for Structural Steel Buildings* (2022).

| Table C-A-7.1<br>Approximate Values of Effective<br>Length Factor, $K$ |      |     |     |     |     |     |
|--|------|-----|-----|-----|-----|-----|
|  | (a)  | (b) | (c) | (d) | (e) | (f) |
| Buckled shape of column is shown by dashed line                        |      |     |     |     |     |     |
| Theoretical $K$ value  | 0.5  | 0.7 | 1.0 | 1.0 | 2.0 | 2.0 |
| Recommended design value when ideal conditions are approximated        | 0.65 | 0.8 | 1.0 | 1.2 | 2.1 | 2.0 |
| End condition code   |      |     |     |     |     |     |

#### Method

Prepare a computational model of six parallel 40-ft long steel W14x82 columns. Separate the columns by 10-ft. for clarity and provide the end restraint conditions shown in Table C-A-7.1. Apply a vertically downward 1-kip load to the top of each column and be sure that this point is not restrained in the vertical direction.

Perform a planar frame (2D) elastic critical load analysis<sup>1</sup> and compute the first 10 buckling modes for the system of columns. View the resulting 10 buckling modes and corresponding critical applied load ratios. Note that you may need to increase the scale of the deflected shape to better observe the buckling modes.

#### Hints:

- 1) Suggested units are kips, inches, and ksi.
- 2) Do not include the self-weight of the member.

<sup>1</sup> An elastic critical load analysis is also referred to as an elastic buckling or eigenvalue analysis.

### MASTAN2 Details

Per Fig. 1, the following suggestions are for those employing MASTAN2 to calculate the above computational strengths:

- ✓ Subdivide the members into 8 elements.
- ✓ In all computational analyses, the failure load will be the product of the applied force (1-kip) and the resulting Applied Load Ratio.

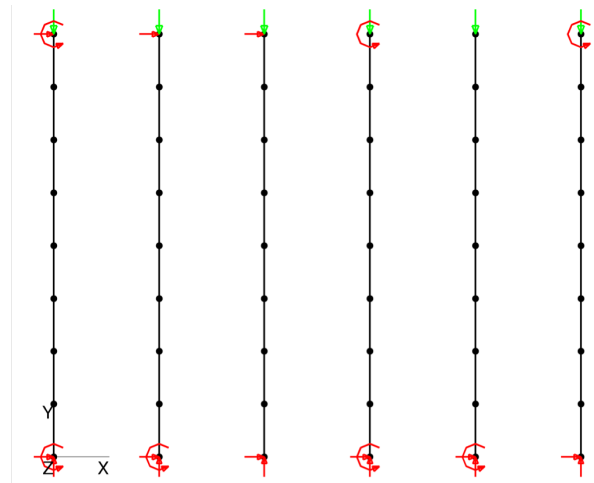


Figure 1. MASTAN2 model.

### Questions

- 1) After recording the buckling loads (product of resulting applied load ratio and the given axial load of 1 kip) for each given end restraint in Table 1, compare these results with the equation  $P_{cr} = \pi^2 EI / (KL)^2$  using the theoretical effective length  $K$ -factors provided in Table C-A-7.1.
- 2) Similarly, back-calculate and compare the effective length  $K$ -factor for each end restraint using  $K = \pi / L \sqrt{EI / P_{cr}}$ , where  $P_{cr}$  is each of the column's critical loads based on computational analysis.
- 3) Sort the buckling loads from smallest to largest. What can be concluded regarding the degree of end restraint in relation to the magnitude of column buckling load?
- 4) In reviewing the 10 buckling modes, are higher buckling modes shown for any of the columns? Why are these modes of little to no interest? When might they be useful?
- 5) Comment on how the actual end restraint may differ from the idealized ones given. Why is your response difficult to model using computational analysis? How would these differences impact the  $K$ -factors and in turn, the corresponding critical loads?

Table 1.

| Column |                | Theoretical<br>$P_{cr}$ | Analysis<br>$P_{cr}$ | %<br>difference | Theoretical<br>$K$ | Analysis<br>$K$ | %<br>difference | Sort<br>Order |
|--------|----------------|-------------------------|----------------------|-----------------|--------------------|-----------------|-----------------|---------------|
| Case   | End Restraints |                         |                      |                 |                    |                 |                 |               |
| a      | fixed—fixed    |                         |                      |                 |                    |                 |                 |               |
| b      | fixed—pinned   |                         |                      |                 |                    |                 |                 |               |
| c      | pinned—pinned  |                         |                      |                 |                    |                 |                 |               |
| d      | fixed—no rot.  |                         |                      |                 |                    |                 |                 |               |
| e      | fixed—free     |                         |                      |                 |                    |                 |                 |               |
| f      | pinned—no rot. |                         |                      |                 |                    |                 |                 |               |

### More Fun with Computational Analysis!

- 1) At the top and bottom of the Euler column (pinned at the bottom and roller at the top), provide additional 10-ft long horizontal steel beams to the right of the column. Provide a roller (vertical restraint) at the free ends of the beams.
  - a. Using the same bending moment of inertia ( $I$ ) for both beams, vary the value of  $I$  from small to large and compute the resulting buckling load for this column. Prepare a two-dimensional plot, with the value of the beams'  $I$  on the horizontal axis and the column's critical buckling load on the vertical axis. In addition to or alternatively, prepare a similar plot with the back-calculated  $K$ -factor on the vertical axis (instead of the critical buckling load). In general, discuss the impact that the flexural stiffness of the beams has on the critical buckling load of the column.
  - b. Repeat the above exercise, but no longer require that the top and bottom beams have equal values for their moment of inertia  $I$ . Perform many critical load analyses for various combinations of top- and bottom-beam  $I$  values. Take advantage of the fact that that the problem is symmetric; that is, reversing the values of top and bottom  $I$ 's will give the same result. Show the results of this study by preparing a three-dimensional plot, with bottom-beam  $I$  value on a horizontal axis (x-axis), top-beam  $I$  value on the other horizontal axis (y-axis), and the columns critical load and/or  $K$ -factor on the vertical axis (z-axis).
  - c. Repeat either or both of the above exercises, providing beams (again, with rollers at their free ends) at the top and bottom of a column that is pin supported at its base and free to translate at its top (Note: This case without the beams was not one of the originally investigated. Why not?)
- 2) Repeat part or all of the above exercise using the combination of (i) a rigid beam (beam with a very large moment of inertia), and (ii) beam-to-column connections of various rotational restraint (rotational spring stiffness).

### Suggested Warm-up Exercise

Prepare a model that only has one column and the Euler end restraint conditions (pinned at base and roller at top). Starting with one element, compare the buckling load computed by an **elastic critical load analysis** with the theoretical value. Explain why the results are not in close agreement. Subdivide the column into two elements and compare the results. Using an acceptable error of 0.5%, keep repeating this subdivision process until the adequate number of elements is obtained. Complete Table 2 below that shows the number of elements and the corresponding percent error. How many elements are needed? Is this result a function of the degree of end restraint? If so, try some other end conditions and determine which end restraint condition should be used and the corresponding number of elements required for 0.5% accuracy?

Table 2.

| Euler Column (pinned—pinned) |                      |                   |              | Controlling end restraint condition is: |                      |                   |              |
|------------------------------|----------------------|-------------------|--------------|---|----------------------|-------------------|--------------|
| # of elements                | Theoretical $P_{cr}$ | Analysis $P_{cr}$ | % difference | # of elements                           | Theoretical $P_{cr}$ | Analysis $P_{cr}$ | % difference |
| 1                            |                      |                   |              |   |                      |                   |              |
| 2                            |                      |                   |              |   |                      |                   |              |
| 4                            |                      |                   |              |   |                      |                   |              |
| 8                            |                      |                   |              |   |                      |                   |              |
| 16                           |                      |                   |              |   |                      |                   |              |

### Additional Resources

MS Excel spreadsheet: [1\\_ElasticBucklingandSupportConditions.xlsx](#)

LM1 Tutorial Video [9 min]:

<http://www.youtube.com/watch?v=DEI4VRLptKg>

AISC Specification for Structural Steel Buildings and Commentary (2022):

<https://www.aisc.org/publications/steel-standards/aisc-360/>

MASTAN2 software:

<http://www.mastan2.com/>